SIMULATION OF REDSHIFTS IN HYDROGEN BY PERTURBATIONS

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NAVAL POSTGRADUATE SCHOOL
Monterey, California

THESIS

SIMULATION OF REDSHIFTS IN HYDROGEN
BY PERTURBATIONS

by

James Roger Meyer

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yield a redshift and conclusions are given for several types of perturbations. The only perturbation that is found to produce a redshift is a perturbation of the form $H' = r^{-1} f(\phi)$.

Simulation of Redshifts in Hydrogen by Perturbations

by

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requirements for the degree of

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ABSTRACT

The problem of explaining the redshifts in the spectra of quasi-stellar objects is considered by using first-order quantum perturbation theory to examine classes of perturbing Hamiltonians to determine if they could cause a redshift in the spectrum of hydrogen. A method is developed for deriving necessary conditions that a perturbation must satisfy to yield a redshift and conclusions are given for several types of perturbations. The only perturbation that is found to produce a redshift is a perturbation of the form $H' = r^{-1} f(\phi)$.

TABLE OF CONTENTS

I. INTRODUCTION

One of the unsolved problems of astronomy is that of accounting for the surprisingly large redshifts found in the spectra of quasi-stellar objects (quasars). It is generally believed that redshifts can be used as measures of distance to astronomical objects. Assuming this is true, the redshifts of quasars are so large, and their consequent distances are so great, that quasars could only be visible on earth if their luminosities were greater than the luminosities of any other known objects.[1] This, in itself, is not a problem because it is simple enough to conceive of objects brighter than any heretofore known. The problem comes from the variability in brightness of some quasars. A few quasars seem to vary periodically in luminosity over a period of time on the order of weeks or months. One quasar (3C 446) varies in brightness by a factor of about two in times as short as one day.[2] It seems likely that for the properties of any object to vary periodically, the radius of the object should be no larger than the distance some sort of signal could travel during one period. If it is assumed

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[1] There are arguments that indicate that relativistic gravitational redshifts could not be as large as the measured redshifts, though there is no general agreement on this point. See Burbidge, Geoffrey, and Burbidge, Margaret, Quasi-Stellar Objects, W. H. Freeman and Company, 1967.

[2] *Ibid.*, p. 215.

that this "signal" travels at the speed of light and if a period of one month is assumed, quasars must be quite small by astronomical standards. The radius of the Milky Way galaxy, for comparison, is on the order of 50,000 light years. And herein lies the problem. With the present state of knowledge, it is difficult to imagine an object so much smaller than our galaxy that is so very much more luminescent.

A number of suggestions have been made to explain this apparent contradiction. It has, for example, been suggested[3] that the observed redshifts are not real and that they are caused by mistaken identification of spectral lines because of the very low light intensities reaching the earth and the consequent fuzziness of the spectra. This explanation appears decreasingly likely as the number of known quasars with measured redshifts increases.

Some astronomers have speculated that an entirely new physics may be needed to explain the properties of quasars. This "new physics" could be some new line of development which would require new concepts in the same way that new concepts allowed the development of quantum theory on a foundation of classical mechanics. It has even been suggested that quasars may obey a different physics from that which governs this part of the universe.

But these speculations are included here only for completeness. The purpose of this thesis is to examine still

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[3] Proctor, W. G., and Waldner, F., A New Interpretation of the Spectra from Quasi-Stellar Objects, preprint, n.d.

another possibility. That is, that the redshifts are not consequences of the distance or relative motion alone but are primarily the result of some physical condition in the quasar which causes a shift in atomic energy levels. This possibility is examined by use of first-order non-relativistic quantum mechanical perturbation theory for the case of a quasar which is assumed to contain hydrogen. This line of attack is attractive, in part, because of another property of some quasars. A few quasars (3C 273, for example)[4] have a measurable angular size because they appear to be associated with gaseous jets or plumes which extend considerable distances into space and whose cause and nature are unknown. It is a plausible speculation that answers to some of the most interesting questions about quasars may lie in the causes of these jets. Perturbation theory can give some idea of classes of perturbations that could cause the observed redshifts. If such classes of perturbations are found to exist, they could be further examined in an attempt to see if they correspond to physical conditions that could exist in quasars and, if so, to see if they could explain the jets.

Because of the absorption of the atmosphere, only a fraction of the radiation from quasars can be observed on earth. It is conceivable that redshifts exist only in those parts

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[4] Kahn, F. D. and Palmer, H. P., Quasars, Their Importance in Astronomy and Physics, Harvard University Press, 1967.

of quasar spectra that can be observed, but for simplicity, it is assumed that all the radiation from quasars is redshifted and that, if a perturbation fails to yield a redshift in only one spectral line, the perturbation may be rejected.

Before proceeding, the term "redshift" should be carefully defined. A redshift is a shift toward the red, or lower frequencies, of each frequency in a spectrum in such a way that the redshifted frequencies are all constant multiples of the corresponding unshifted frequencies, where the constant is a number between zero and one. If this constant is denoted by S, ν' is the redshifted frequency, ν is the unshifted frequency, E_i and E_k are the unpertubed energy levels, and W_i and W_k are the perturbations of the energy levels caused by some perturbing Hamiltonian which is assumed to be the cause of the redshift, then

$$\nu' = S\nu = S\left(\frac{E_k - E_i}{h}\right)$$

and

$$\nu' = \frac{SE_k - SE_i}{h}.$$

But it must also be true that

$$\nu' = \frac{(E_k + W_k) - (E_i + W_i)}{h}.$$

Hence

$$\frac{(E_k + W_k) - (E_i + W_i)}{h} = \frac{SE_k - SE_i}{h}$$

which is clearly satisfied if

$$W_n = s\,E_n \qquad (1)$$

In order for S to be between zero and one, we must also have

$$-1 < s < 0.$$

Equation (1) must be satisfied by the perturbations of each energy level, for a fixed value of s, in order for a red-shift to exist.

II. SPHERICALLY SYMMETRIC PERTURBATIONS

Because perturbations of the hydrogen atom are being considered, it is necessary to use perturbation theory for the case where degeneracies exist. In this case, the first order perturbations in the energy levels are the solutions, W, of [5,6]

$$\text{Det}\begin{vmatrix} (a_{11}-W) & a_{12} & a_{13} & \cdots & a_{1n^2} \\ a_{21} & (a_{22}-W) & a_{23} & \cdots & a_{2n^2} \\ \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdot & & \cdot \\ a_{n^2 1} & a_{n^2 2} & a_{n^2 3} & \cdots & (a_{n^2 n^2}-W) \end{vmatrix} = 0 \qquad (2)$$

where det indicates determinant, $a_{ij} = \langle n\alpha_i | H' | n\alpha_j \rangle$,n is the principal quantum number of an unperturbed hydrogen atom state, H' is the perturbing Hamiltonian, and α_k stands for whatever additional parameters are necessary to completely specify the unperturbed states of the hydrogen atom.

Suppose the perturbing Hamiltonian is spherically symmetric; that is, suppose $H' = H'(r)$. In this case all the off-diagonal elements in equation (2) vanish. This is true

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[5] Park, David, Introduction to the Quantum Theory, p. 218, McGraw-Hill Book Company, 1964.

[6] Pauling, Linus and Wilson, E. Bright, Introduction to Quantum Mechanics, McGraw-Hill Book Company, 1935.

because

$$a_{ij} = \langle n\alpha_i | H'(r) | n\alpha_j \rangle$$

$$= \int u^*_{n\alpha_i} \, H'(r) \, u_{n\alpha_j} \, d^3r$$

$$= \int R^*_{nl_i}(r) \, Y^*_{l_i m_i}(\theta,\phi) \, H'(r) \, R_{nl_j}(r) \, Y_{l_j m_j}(\theta,\phi) \, d^3r$$

$$= \int_0^\infty dr \, R^*_{nl_i}(r) \, H'(r) \, R_{nl_j}(r) \, r^2 \int_0^\pi d\theta \sin\theta \int_0^{2\pi} d\phi \, Y^*_{l_i m_i}(\theta,\phi) \, Y_{l_j m_j}(\theta,\phi)$$

But, because of the orthogonality of $Y_{lm}(\theta,\phi)$

$$\int_0^\pi d\theta \sin\theta \int_0^{2\pi} d\phi \; Y^*_{l_i m_i}(\theta,\phi) \; Y_{l_j m_j}(\theta,\phi) = 0 \qquad \text{for } i \neq j.$$

Hence $a_{ij} = 0$ if $i \neq j$ and $H' = H'(r)$.

Since all the off-diagonal elements are zero, it is clear that the solutions to equation (2) are

$$W = a_{kk} \qquad k = 1, 2, 3, \cdots, n^2.$$

If the n^2 values of W are not all equal, the n^{th} energy level will be split and there will be no redshift. Consequently, it must be true that the roots of equation (2) are all equal if a redshift is to exist. It may be true, however, that the values of W are not all equal but that they are nearly equal so that the resulting line splitting might not be

visible in the fuzzy spectra obtained from quasars. The result could be that a redshift could appear to exist even if equation (1) is not satisfied for all roots of equation (2). For this reason, it is not required here that all the roots of equation (2) be equal when spherically symmetric perturbations are considered. Allowing the roots of equation (2) to be unequal is possible because, for spherically symmetric perturbations, the off-diagonal elements of equation (2) vanish making the solutions of equation (2) relatively simple to find. This would not be the case for a non-spherically symmetric perturbation.

A useful result for spherically symmetric perturbing Hamiltonians is the following theorem:

> Theorem: If H' is spherically symmetric, there can be, at most, n distinct roots of equation (2).
>
> Proof:

$$W = \langle n\alpha_k | H'(r) | n\alpha_k \rangle \qquad k = 1, 2, 3, \cdots, n^2$$

$$= \int_0^\infty dr\, |R_{n\ell}(r)|^2 H'(r) r^2 \int_0^\pi d\theta \sin\theta \int_0^{2\pi} d\phi\, |Y_{\ell m}(\theta,\phi)|^2 .$$

But, if $Y_{\ell m}(\theta,\phi)$ is normalized,

$$\int_0^\pi d\theta \sin\theta \int_0^{2\pi} d\phi\, |Y_{\ell m}(\theta,\phi)|^2 = 1.$$

Hence

$$W = \int_0^\infty dr\, |R_{n\ell}(r)|^2 H'(r) r^2 \qquad (3)$$

and W is independent of the quantum number m. Since n is held fixed, W is a function only of the quantum number ℓ and there are n distinct values of ℓ . Hence, there are at most n distinct roots, W, of equation (2)

Consider now perturbations of the form

$$H' = A_k r^{-k} \tag{4}$$

where k is a non-negative integer. To determine whether or not such perturbing Hamiltonians can produce redshifts, one must calculate the values of a_{ij} and check the resulting values of W to see if they satisfy equation (1).

The case k = 0 is trivial. In that case $H' = A_0$ = constant, so that for n = 1

$$W = \langle n\alpha_1 | A_0 | n\alpha_1 \rangle = A_0 \langle n\alpha_1 | n\alpha_1 \rangle = A_0 \tag{5}$$

assuming the wave function is normalized. From equation (1)

$$s = \frac{A_0}{E_1} .$$

For n = 2,

$$W = \langle n\alpha_1 | A_0 | n\alpha_1 \rangle = \langle n\alpha_2 | H' | n\alpha_2 \rangle = \langle n\alpha_3 | H' | n\alpha_3 \rangle = \langle n\alpha_4 | H' | n\alpha_4 \rangle = A_0 . \tag{6}$$

From equation (1)

$$s = \frac{A_0}{E_2} .$$

Both values of s must be equal for a redshift to exist but they cannot be equal in this case because

$$E_n = \frac{E_1}{n^2} . \tag{7}$$

Therefore there can be no redshift for a perturbation $H' = A_0$. This could have been predicted from equations (5) and (6) because it is clear that this perturbation will change all energy levels by the fixed amount, A_0, and will therefore have no observable effect.

The next perturbation to be examined is

$$H' = A_1 r^{-1}.$$

Clearly, when this perturbing Hamiltonian is added to the unperturbed Hamiltonian, it has only the effect of multiplying the existing Coulomb term in the unperturbed Hamiltonian by a constant and the result is the same as if, for example, the charge on the electron were changed. It may be obvious that this could result in a redshift but there is a simple way this can be demonstrated. We know that

$$E_n = C e^4$$

where

$$C = \frac{-\mu}{32\pi^2 \epsilon_0^2 \hbar^2 n^2}.$$

and μ is the reduced mass of the electron in the hydrogen atom. Then

$$dE_n = 4C e^3 de.$$

Hence

$$dE_n = \frac{4\,de}{e} E_n.$$

Since $\frac{4\,de}{e}$ is independent of n, equation (1) is satisfied for all n provided

$$-1 < \frac{4\,de}{e} < 0$$

and this perturbation yields a redshift. But since this perturbation amounts only to a change in the electron charge, it is clearly a trivial example of a perturbation that yields a redshift.

Next, consider a perturbation

$$H' = A_2 r^{-2}.$$

Using the known, unperturbed hydrogen atom wave functions[7], yields, for n = 1

$$W = \langle n\alpha_1 | H' | n\alpha_1 \rangle$$

$$= \frac{A_2}{\pi a_0^3} \int e^{-\frac{2r}{a_0}} \sin\theta \, dr \, d\theta \, d\phi$$

$$= \frac{2A_2}{a_0^2}$$

where a_0 is the Bohr radius, $\frac{4\pi \epsilon_0 \hbar^2}{\mu e^2}$. Since

$$E_n = \frac{-\mu e^4}{32\pi^2 \epsilon_0^2 \hbar^2 n^2},$$

one can write

$$\frac{1}{a_0^2} = \frac{-2\mu n^2 E_n}{\hbar^2} .$$

Therefore,

$$W = \frac{-4 A_2 \mu}{\hbar^2} E_1 .$$

and equation (1) gives

$$s = \frac{-4A_2 \mu}{\hbar^2}.$$

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[7]Beiser, Arthur, Perspectives of Modern Physics, p. 202, McGraw-Hill Book Company, 1969.

For n = 2, 1 = 0

$$W = \frac{A_2}{16}\left(\frac{1}{2\pi a_0^3}\right)\int\left(2 - \frac{r}{a_0}\right)^2 e^{-\frac{r}{a_0}} \sin\theta \, dr \, d\theta \, d\phi$$

$$= \frac{A_2}{2a_0^2}$$

$$= \frac{-4A_2\mu}{\hbar^2} E_2 .$$

This value of W yields a value of s equal to the value of s for n = 1, so, at first glance, it appears that this perturbation may yield a redshift. When the values of W for n = 3 are calculated, however, one sees that there can be no redshift. For n = 3, 1 = 0

$$W = \frac{A_2}{81}\left(\frac{1}{3\pi a_0^3}\right)\int\left(\frac{2r^2}{9a_0^2} - \frac{2r}{a_0} + 3\right)^2 e^{-\frac{2r}{3a_0}} \sin\theta \, dr \, d\theta \, d\phi$$

$$= \frac{-4A_2\mu}{3\hbar^2} E_3 .$$

For n = 3, 1 = 1

$$W = \frac{A_2}{729}\left(\frac{2}{\pi a_0^5}\right)\int\left(2 - \frac{r}{3a_0}\right)^2 e^{-\frac{2r}{3a_0}} r^2 \cos^2\theta \sin\theta \, dr \, d\theta \, d\phi$$

$$= \frac{-4A_2\mu}{9\hbar^2} E_3 .$$

For n = 3, 1 = 2

$$W = \frac{A_2}{6561}\left(\frac{1}{6\pi a_0^7}\right)\int e^{-\frac{2r}{3a_0}} r^4 (3\cos^2\theta - 1)^2 \sin\theta \, dr \, d\theta \, d\phi$$

$$= \frac{-4A_2\mu}{15\hbar^2} E_3 ,$$

and none of the three possible distinct values of W yields a value of s equal to the value of s for n = 1. Consequently, this perturbation does not lead to a redshift.

Perturbations of the form

$$H' = A_k r^{-k} \qquad k \geq 3$$

can be treated in the following way. It is known that the radial part of the unperturbed hydrogen wave function is[8]

$$R_{n\ell}(\rho) = A_{n\ell}\,\rho^{\ell} e^{-\frac{\rho}{2}}\, L^{(2\ell+1)}_{(n+\ell)}(\rho) \qquad \rho = \frac{2r}{n a_0}$$

where

$$L^{(2\ell+1)}_{(n+\ell)}(\rho) = -\frac{(n+\ell)!}{(n-\ell-1)!}\, e^{\rho}\, \rho^{-2\ell-1}\, \frac{d^{(n-\ell-1)}}{d\rho^{(n-\ell-1)}}\left(e^{-\rho}\rho^{n+\ell}\right).$$

Clearly,

$$\frac{d^{(n-\ell-1)}}{d\rho^{(n-\ell-1)}}\left(e^{-\rho}\rho^{n+\ell}\right)$$

will be a polynomial in ρ whose highest power of ρ will be $n + \ell$. Then the highest power of ρ in $L^{(2\ell+1)}_{(n+\ell)}(\rho)$ will be $n-\ell-1$. Similarly, the highest power of ρ, and consequently of r, in $R_{n\ell}(\rho)$ will be n-1 and the highest power of r in $|R_{n\ell}(r)|^2 r^{-k} r^2$ will be 2n-k. That is, the highest power of r in the integral of equation (3) will be 2n-k. Hence, for each odd value of k greater than two, there will be a value of n, specifically $\frac{k-1}{2}$, such that

$$2n - k = -1$$

which means that equation (3) will contain a term of the form

$$\int_0^{\infty} r^{-1} e^{-ar}\, dr \qquad (a>0).$$

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[8]Schiff, Leonard I., Quantum Mechanics, 3d ed., p. 93, McGraw-Hill Book Company, 1968.

But[9]

$$\int_0^{\infty} r^{-1} e^{-ar}\, dr = \infty. \tag{8}$$

The same problem exists for every value of k greater than two. To see this, note that for any value of ℓ except n-1 (and such a value of ℓ is always possible for $n > 1$),

$$\frac{d^{(n-\ell-1)}\left(e^{-\rho}\rho^{n+\ell}\right)}{d\rho^{(n-\ell-1)}}$$

will contain a term in ρ whose power is n-ℓ-1 and the corresponding term in $R_{n\ell}(r)$ will contain r raised to the n-2 power. Then, when $|R_{n\ell}(r)|$ is squared, the product of the terms containing r^{n-1} and r^{n-2} will be a term containing r^{2n-3} so that the corresponding term in $|R_{n\ell}(r)|^2 r^{-k} r^2$ will contain r^{2n-1-k}. By the same process as above, it is clear that for every even value of k greater than two, there is a value of n, specifically $\frac{k}{2}$, such that W will contain a term of the form in equation (8).

The divergence of W for perturbing Hamiltonians of the form (4), with $k \geq 3$, is, of course, a consequence of the singularity of these Hamiltonians at the origin. If these Hamiltonians are to represent real physical processes, then presumably the singularities are "cut off" at small distances, thereby yielding finite energy shifts. However, as they stand, these Hamiltonians are clearly untenable.

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[9]Jahnke, Eugene and Emde, Fritz, Tables of Functions, 4th ed., pp. 1-6, Dover Publications, 1945.

Now consider spherically symmetric perturbing Hamiltonians of the form

$$H' = B_k r^k \qquad k = 1, 2, 3, \cdots \tag{9}$$

Each of these perturbations implies that the perturbation is one which becomes greater for increasing values of r. However, one should note that functions of r exist whose values decrease with increasing r and which can be written as linear combinations of terms of the form (9). An example is

$$e^{-\frac{r}{r_0}} = 1 - \frac{r}{r_0} + \frac{1}{2!}\left(\frac{r}{r_0}\right)^2 - \frac{1}{3!}\left(\frac{r}{r_0}\right)^3 + \cdots \qquad r_0 = \text{constant.}$$

Consider the general perturbation of the form of equation (9). For n = 1

$$W = \frac{B_k}{\pi a_0^3}\int e^{-\frac{2r}{a_0}} r^{2+k} \sin\theta \, dr \, d\theta \, d\phi$$

$$= \frac{(2+k)! \; a_0^k B_k}{2^{k+1}} .$$

From equation (1)

$$s = \frac{(2+k)! \; a_0^k B_k}{2^{k+1} \; E_1} .$$

For n = 2, l = 0

$$W = \frac{B_k}{16}\left(\frac{1}{2\pi a_0^3}\right)\int \left(2 - \frac{r}{a_0}\right)^2 e^{-\frac{r}{a_0}} r^{2+k} \sin\theta \, dr \, d\theta \, d\phi$$

$$= B_k a_0^k \left[\frac{1}{2}(2+k)! - \frac{1}{2}(3+k)! + \frac{1}{8}(4+k)!\right]$$

and equation (1) yields

$$s = \frac{B_k a_0^k}{E_2}\left[\tfrac{1}{2}(2+k)! - \tfrac{1}{2}(3+k)! + \tfrac{1}{8}(4+k)!\right].$$

Using equation (7) and setting the two values of s equal to each other gives

$$\frac{(2+k)!\; a_0^k B_k}{2^{3+k} E_2} = \frac{B_k a_0^k}{E_2}\left[\tfrac{1}{2}(2+k)! - \tfrac{1}{2}(3+k)! + \tfrac{1}{8}(4+k)!\right]$$

which reduces to

$$\frac{1}{2^{3+k}} = \tfrac{1}{8}k^2 + \tfrac{3}{8}k + \tfrac{1}{2}. \qquad (10)$$

But equation (10) clearly is not satisfied for any positive integral value of k. Hence, this value of W does not produce a redshift. The other value of W, for n = 2, 1 = 1, is

$$W = \frac{B_k}{16 a_0^2}\left(\frac{1}{2\pi a_0^3}\right)\int e^{-\frac{r}{a_0}} r^{4+k} \cos^2\theta \sin\theta \, dr\, d\theta\, d\phi$$

$$= \frac{(4+k)!}{24} B_k a_0^k.$$

If there is to be a redshift, this value of W must yield the same value of s as was obtained for n = 1. Using equations (7) and (1)

$$\frac{(4+k)!}{6}\,\frac{B_k a_0^k}{E_1} = \frac{(2+k)!\; a_0^k B_k}{2^{1+k} E_1}$$

which reduces to

$$12 + 7k + k^2 = \frac{3}{2^k}.$$

But this equation is clearly not satisfied for any positive integral value of k. Hence, no perturbation of the form of equation (9) can produce a redshift and no perturbation of

the form of equation (4) yields a redshift for any integral value of k except the trivial case of k = 1.

If a perturbation is of the form

$$H' = \sum_{k=-2}^{+\infty} A_k r^k \qquad (11)$$

it may well be possible, because of the infinite number of arbitrary constants A_k, to obtain a "redshift" for a finite number of spectral lines. It is not at all clear, however, that a perturbation of the form of (11) could yield a redshift for all the infinite number of lines in a spectrum.

III. NON-SPHERICALLY SYMMETRIC PERTURBATIONS

The remaining problem is to examine the case where H' is not spherically symmetric, that is, H' has some angular dependence. If no simplifying assumptions were made, it would be necessary to examine angular dependent perturbations by solving equation (2) for the case when the off-diagonal elements do not necessarily vanish. But a theorem can be proved that shows that W is independent of the off-diagonal elements if all roots of equation (2) are required to be equal, that is if the perturbation is not to break the degeneracy. As was stated above, this condition is, strictly speaking, necessary for a redshift.

Theorem: If all the roots of equation (2) are equal, they are given by

$$W = \frac{1}{n^2}\left(a_{11} + a_{22} + a_{33} + \cdots + a_{n^2 n^2}\right).$$

Proof: Equation (2) is an n^2 order polynomial in the unknown, W. If the roots are denoted by $R_1, R_2, R_3, \cdots, R_{n^2}$, then the equation can be re-written as

$$(W-R_1)(W-R_2)(W-R_3)\cdots(W-R_{n^2}) = 0$$

It is clear that, if the left hand side of this equation is multiplied out, the coefficient of the W^{n^2-1} term will be $-(R_1+R_2+R_3+\cdots+R_{n^2})$. If all the roots of the equation are constrained to be equal, then $R=R_1=R_2=\cdots=R_{n^2}$, so that the

coefficient of the W^{n^2-1} term will be $-n^2R$.
It is also simple to demonstrate that the same coefficient in the n^2 order equation that is obtained by expanding equation (2) is $-(a_{11}+a_{22}+\cdots+a_{n^2n^2})$.
Setting these two coefficients equal yields

$$-n^2R = -(a_{11} + a_{22} + a_{33} + \cdots + a_{n^2n^2}) .$$

Hence

$$W = R = \frac{1}{n^2}(a_{11} + a_{22} + a_{33} + \cdots + a_{n^2n^2}).$$

An angularly dependent perturbation which is obviously of interest is $H' = Ar\cos\theta$ which is the form of perturbation that would exist in a constant force field where the atom is assumed to be aligned so that the perturbing Hamiltonian is independent of ϕ . For n = 1

$$W = \frac{A}{\pi a_0^3}\int e^{-\frac{2r}{a_0}} r^3 \sin\theta \cos\theta \, dr\, d\theta\, d\phi .$$

But

$$\int_0^\pi \cos^j\theta \sin\theta \, d\theta = 0 \qquad j = 1,3,5,\ldots$$

Consequently, for n = 1, W = 0 and, since the ground state energy level is not perturbed, there can be no redshift. Clearly, this is also true for any perturbation of the form

$$H' = f(r,\phi)\cos^j\theta \qquad j = 1,3,5,\ldots$$

By the same process, since

$$\int_0^\pi \sin(a\theta)\sin\theta \, d\theta = 0 \qquad (a \text{ an integer} \neq 1)$$

there can be no redshift for any perturbation of the form

$$H' = f(r, \phi) \sin(a\theta) \qquad \text{(a an integer } \neq 1\text{)}$$

Suppose a perturbation has the form

$$H' = f(r, \theta)\, g(\phi) \tag{12}$$

Let

$$\int_0^{2\pi} g(\phi)\, d\phi = C \tag{13}$$

Since the ϕ dependence of $u_{n\ell m}(r,\theta,\phi)$ is $e^{im\phi}$, $|u_{n\ell m}(r,\theta,\phi)|^2$ is independent of ϕ so that the ϕ integral for every value of a_{ii} will be the integral of equation (13). That is, every value of W will have the same constant factor C. Hence the ϕ dependence of equation (12) will have no effect on the existence or non-existence of a redshift. The only effect of a separable ϕ dependence in a perturbation will be in the magnitude of any redshift that does exist. Note that if

$$H' = f(r, \theta, \phi)$$

and if the perturbation cannot be separated into the form of equation (12), then the ϕ dependence may determine whether or not there is a redshift.

Consider a perturbation which is a function of θ alone. That is, $H' = f(\theta)$. For n = 1

$$W = a_{11} = \frac{1}{\pi a_0^3} \int e^{-\frac{2r}{a_0}} f(\theta)\, r^2 \sin\theta \, dr\, d\theta\, d\phi$$

$$= \frac{1}{2} \int_0^{\pi} f(\theta) \sin\theta \, d\theta$$

$$s = \frac{1}{2E_1} \int_0^{\pi} f(\theta) \sin\theta \, d\theta .$$

For n = 2, if H' does not break the degeneracy of the n = 2 state,

$$a_{11} = \frac{1}{32\pi a_0^3} \int \left(2 - \frac{r}{a_0}\right)^2 e^{-\frac{r}{a_0}} f(\theta) \sin\theta \, r^2 \, dr \, d\theta \, d\phi$$

$$= \frac{1}{2} \int_0^{\pi} f(\theta) \sin\theta \, d\theta$$

$$a_{22} = \frac{1}{32\pi a_0^5} \int r^4 e^{-\frac{r}{a_0}} \cos^2\theta \, f(\theta) \sin\theta \, dr \, d\theta \, d\phi$$

$$= \frac{3}{2} \int_0^{\pi} f(\theta) \cos^2\theta \sin\theta \, d\theta$$

$$a_{33} = a_{44} = \frac{1}{64\pi a_0^5} \int r^4 e^{-\frac{r}{a_0}} f(\theta) \sin^3\theta \, dr \, d\theta \, d\phi$$

$$= \frac{3}{4} \int_0^{\pi} f(\theta) \sin^3\theta \, d\theta .$$

Then

$$W = \frac{1}{4} \left(a_{11} + a_{22} + a_{33} + a_{44}\right)$$

$$= \frac{1}{2} \int_0^{\pi} f(\theta) \sin\theta \, d\theta .$$

Hence

$$s = \frac{1}{2E_2} \int_0^{\pi} f(\theta) \sin\theta \, d\theta .$$

Since $E_1 \neq E_2$, these two values of s are equal only if the integral is zero. But if the integral is zero, then the perturbation, W, is zero for n = 1 or n = 2. Hence, there can be no redshift for $H' = f(\theta)$. Since a separable ϕ dependence does not affect the existence or non-existence of

a redshift, as proved above, there can also be no redshift for a perturbation of the form

$$H' = f(\theta)\, g(\phi).$$

It is possible to derive a necessary condition for any perturbation to yield a redshift. This can be done by requiring that H' does not break the degeneracy of the n = 2 states and writing the general expression for the values of W(n = 1) and W(n = 2). Then using equation (1), the corresponding values of s can be determined. Setting these equal, as they must be for a redshift, yields a condition required of H' for a redshift to exist. For n = 1

$$W = \frac{1}{\pi a_0^3} \int e^{-\frac{2r}{a_0}} H' \, d^3r$$

$$s = \frac{1}{\pi a_0^3 E_1} \int e^{-\frac{2r}{a_0}} H' \, d^3r$$

For n = 2,

$$W = \frac{1}{4}(a_{11} + a_{22} + a_{33} + a_{44})$$

$$= \frac{1}{4}\left[\frac{1}{32\pi a_0^3}\int \left(2 - \frac{r}{a_0}\right)^2 e^{-\frac{r}{a_0}} H' d^3r + \frac{1}{32\pi a_0^5}\int r^2 e^{-\frac{r}{a_0}} \cos^2\theta \, H' d^3r \right.$$
$$\left. + \frac{1}{64\pi a_0^5}\int r^2 e^{-\frac{r}{a_0}} \sin^2\theta \, H' d^3r + \frac{1}{64\pi a_0^5}\int r^2 e^{-\frac{r}{a_0}} \sin^2\theta \, H' d^3r \right].$$

Using equations (1) and (7)

$$s = \frac{1}{E_1}\left[\frac{1}{32\pi a_0^3}\int \left(2 - \frac{r}{a_0}\right)^2 e^{-\frac{r}{a_0}} H' d^3r + \frac{1}{32\pi a_0^5}\int r^2 e^{-\frac{r}{a_0}} \cos^2\theta \, H' d^3r \right.$$
$$\left. + \frac{1}{32\pi a_0^5}\int r^2 e^{-\frac{r}{a_0}} \sin^2\theta \, H' d^3r \right].$$

Setting these two values of s equal and simplifying yields

$$\int \left(e^{-\frac{r}{a_0}} - \frac{1}{8} + \frac{r}{8a_0} - \frac{r^2}{16a_0^2} \right) e^{-\frac{r}{a_0}} H' d^3r = 0 . \tag{14}$$

Equation (14) is a necessary condition on H' that must be true for a redshift to exist. The same procedure can be followed to develop other relations which must be true of H' for a redshift to exist. It is only necessary to require that H' not break the degeneracies of the n = j and n = k states, then find the values of s for n = j and n = k. Equating these values of s gives an additional necessary condition on H' for a redshift to exist. An infinite number of these conditions can be obtained, but for higher values of n, the expressions become increasingly complicated and decreasingly useful as a tool for examining possible perturbations. Nevertheless, the existence of these necessary conditions would appear to rather severly restrict the possibilities of obtaining a redshift by means of a perturbing Hamiltonian.

It can be shown easily that the trivial case, $H' = A_1 r^{-1}$, satisfies equation (14) as does a perturbation of the form

$$H' = r^{-1} f(\theta, \phi). \tag{15}$$

Presumably, the additional expressions which could be derived by equating values of s for other pairs of energy levels would not all be satisfied by a perturbation of the form (15). But a little reflection will show that all the additional

expressions would be satisfied by a perturbation of the form

$$H' = r^{-1} f(\phi)$$

as might be expected from the earlier conclusion about perturbations with a separable ϕ dependence.

IV. CONCLUSIONS

The large redshifts in the spectra of quasars and the periodic variations in luminosities of some quasars are difficult to understand because they seem to require contradictory conclusions about quasars. It has been the purpose of this thesis to examine the possibility that quasars are not at cosmological distances, as their redshifts suggest, but that the redshifts are caused by some physical condition in the quasar causing a perturbation in the hydrogen energy levels.

A number of possible perturbing Hamiltonians were examined. In all cases where a conclusion could be reached, except one, it was found that the perturbation could not produce a redshift. The exception was the trivial case of the $\frac{1}{r}$ perturbation. Judging by these results, it appears that the production of a redshift by a perturbing Hamiltonian is somewhat less likely than might have been expected.

Specifically, none of the following perturbations were found to produce a redshift:

$H' = A_k r^{-k}$ $k = 2,3,4,\ldots$

$H' = B_k r^{k}$ $k = 0,1,2,3,\ldots$

$H' = f(r,\phi) \cos^j \theta$ $j = 1,3,5,\ldots$

$H' = f(r,\phi) \sin(a\theta)$ a an integer $\neq 1$

$H' = f(\theta)\, g(\phi).$

A perturbation with a separable ϕ dependence would yield a redshift if, and only if, the ϕ independent part of the perturbation would, by itself, produce a redshift.

Additionally,

$$\int \left(e^{-\frac{r}{2a_0}} - \frac{1}{8} + \frac{r}{8a_0} - \frac{r^2}{16a_0^2}\right) e^{-\frac{r}{a_0}} H' d^3r = 0$$

was found to be a necessary condition on H' if H' were to produce a redshift, and a method of deriving any number of additional necessary conditions on H' was presented.

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